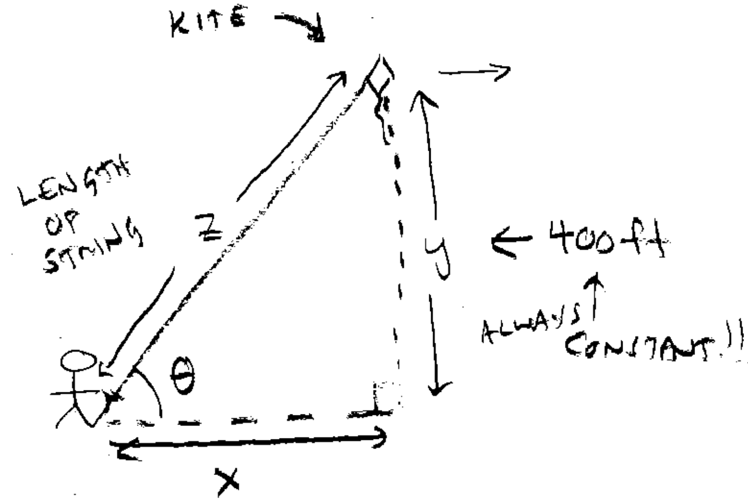
Example: (Like HW 3.9/3)

A kite in the air at an altitude of 400 ft is being blown horizontally at the rate of 10 ft/sec away from the person holding the kite string at ground level.

At what rate is the string being let out when 500 ft of string is already out?



$x = x(t)$ = horizontal distance
$z = z(t)$ = amount of string let out

KNOW: $\frac{dx}{dt} = 10 \ \frac{ft}{sec}$

WANT: $\frac{dz}{dt} = ??? \ \frac{ft}{sec}$ WHEN $z = 500$ ft

$$x^2 + 400^2 = z^2 \Rightarrow 2x\frac{dx}{dt} + 0 = 2z\frac{dz}{dt}$$

$$\Rightarrow \underset{???}{x}\ \underset{10}{\frac{dx}{dt}} = \underset{500}{z}\ \frac{dz}{dt} \Rightarrow 300 \cdot 10 = 500 \cdot \frac{dz}{dt}$$

$$6 = \frac{dz}{dt}$$

$$\boxed{6 \ ft/sec}$$

SOLVE FOR X NOW

$x^2 + 400^2 = 500^2$

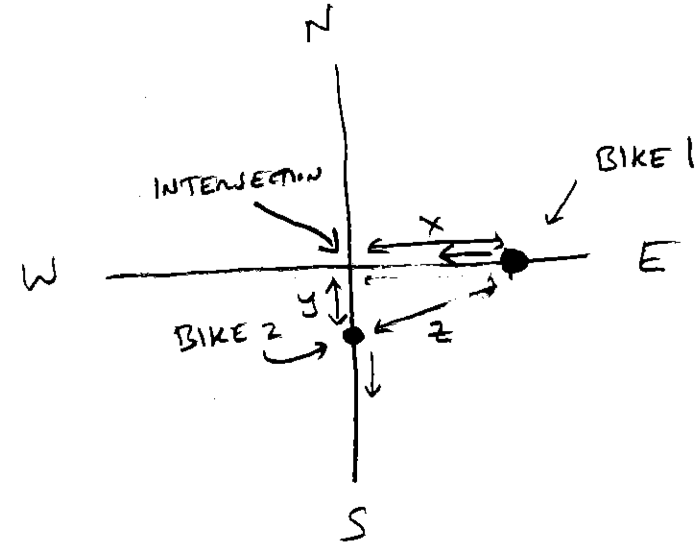
$\Rightarrow x = 300$

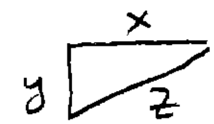
Example: (Like HW 3.9/2)

One bike is 4 miles east of an intersection, travelling toward the intersection at the rate of 9 mph.

At the same time, a 2nd bike is 3 miles south of the intersection and is travelling away from the intersection at a rate of 10 mph.

- At what rate is the distance between them changing?
- Is this distance increasing or decreasing?





KNOW: $\frac{dx}{dt} = -9$ WHEN $x = 4$

$\frac{dy}{dt} = 10$ WHEN $y = 3$

WANT: $\frac{dz}{dt} = ???$ WHEN $x=4, y=3$

$$z^2 = x^2 + y^2$$

$$\Rightarrow 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$\underset{???}{z}\frac{dz}{dt} = \underset{4}{x}\underset{(-9)}{\frac{dx}{dt}} + \underset{3}{y}\underset{(10)}{\frac{dy}{dt}}$$

$z^2 = x^2 + y^2$

$z^2 = 4^2 + 3^2 \Rightarrow z = 5$

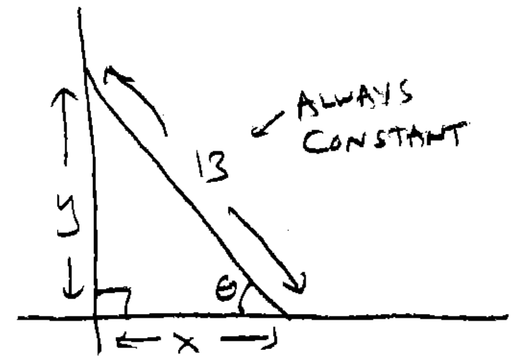
$5\frac{dz}{dt} = -36 + 30 = -6$

$$\boxed{\frac{dz}{dt} = -\frac{6}{5} \text{ mph}}$$

DECREASING!!

Example: (Like 3.6-9/13, 3.9/9)

A 13-foot ladder is leaning against a wall and its base is slipping away from the wall at a rate of 3 ft/sec when it is 5 ft from the wall.



How fast is the top of the ladder dropping at that moment?

KNOW : $\frac{dx}{dt} = 3$ WHEN $x = 5$

WANT : $\frac{dy}{dt} = ?$ WHEN $x = 5$

$$x^2 + y^2 = 13^2 = 169$$

$$\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0 \quad \Rightarrow \quad (5)(3) + (12)\frac{dy}{dt} = 0$$

$x \uparrow 5$, $\frac{dx}{dt} \uparrow 3$, $y \uparrow$???

$$5^2 + y^2 = 13^2 \Rightarrow y^2 = 169 - 25 = 144$$

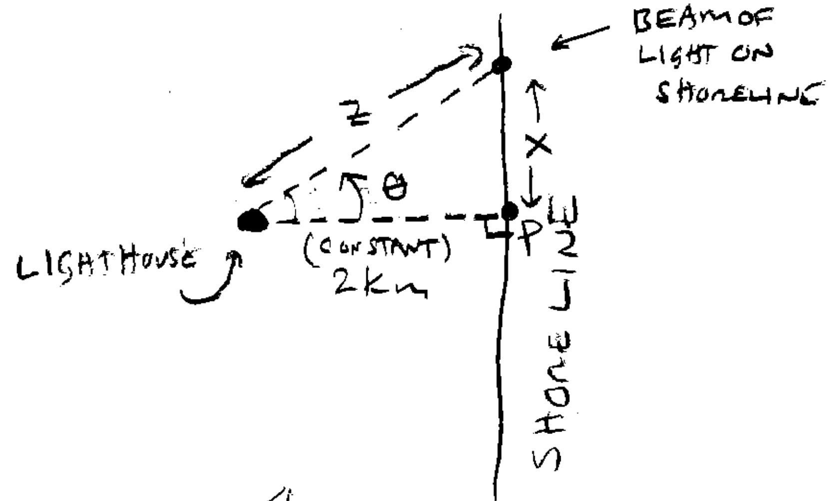
$$y = 12$$

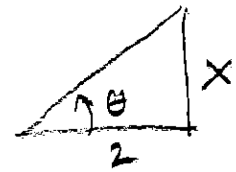
$$\Rightarrow \boxed{\frac{dy}{dt} = -\frac{15}{12} = -\frac{5}{4} = -1.25 \text{ ft/sec}}$$

Example: (Like 3.9/6)

A lighthouse is located on a small island 2 km away from the nearest point *P* on a straight shoreline and its light makes three revolutions per minute.

How fast is the beam of light moving along the shoreline when it is 1 km from *P*?





KNOW: $\frac{d\theta}{dt} = \frac{3 \text{ REV}}{\text{MIN}} = \frac{6\pi \text{ RAD}}{\text{MIN}}$

WANT: $\frac{dx}{dt} = ???$ WHEN $x = 1$ km

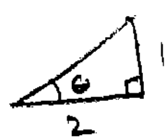
$\tan\theta = \frac{x}{2} \Rightarrow x = 2\tan\theta \Longrightarrow \frac{dx}{dt} = 2\underbrace{\sec^2\theta}_{???}\ \frac{d\theta}{dt}$

WHEN $x=1$ → ???, 6π → $\frac{d\theta}{dt}$

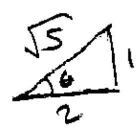
$= 2\left(\frac{\sqrt{5}}{2}\right)^2 6\pi = 2\,\frac{5}{4}\cdot 6\pi$

$= \boxed{15\pi \approx 47.124 \text{ km/min}}$

$x = 1 \Rightarrow 1 = 2\tan\theta$
$\Rightarrow \frac{1}{2} = \tan\theta$



$\Rightarrow$



$\Rightarrow \cos\theta = \frac{2}{\sqrt{5}}$

$\Rightarrow \sec\theta = \frac{\sqrt{5}}{2}$